# Response to Reviewers’ Comments

We would like to thank the editor and reviewers for their valuable comments and suggestions. The comments and suggestions have significantly helped improve the quality of this manuscript. All the reviewers’ comments are fully addressed in the new manuscript. Especially, we have clarified the novelty and technical contributions of this work. We have also improved the theorem proofs to make them more rigorous and easier to understand. Besides, we add a new set of simulation (not done yet). The revisions are highlighted by the texts in orange color in the revised manuscript. The following paragraphs detail our responses to each reviewer’s comments.

# Reviewer #1

1. **The simulation results are provided and some comparison results are provided. But this part should be improved to justify the claims made in the introduction. As the target distribution is not known in advance, the proposed Bayesian filter is also an approximation. How good it is when compared with parameterized particle filter?**

**Response:**

Run the particle filter. If the result is better than the histogram filter, then mention it in the paper. If not, show results here to tell the reviewer.

The authors would like to thank the reviewer for the comment. In general, the Bayesian filter is an optimal and generic framework for nonlinear filtering \cite. Many popular filtering approaches, including the Kalman filter and particle filter, can be derived from the Bayesian filter. It is correct that the target distribution is unknown in advance. However, the estimation results from the Bayesian filter will rely less and less on the initial target distribution, which is proved in Section V. Therefore, even the target distribution is unknown in advance, we can still use the Bayesian filter to do the estimation, just as a particle filter will do. And in this case the Bayesian filter will not be an approximation, but an exact method to fuse sensor measurements.

There are different practical ways to implement the Bayesian filter. The particle filter is a popular way. The other way is the histogram filter, which is used in this manuscript for the simulation. The histogram filter is advantageous by keeping track of the probability mass over the whole field. The particle filter is advantageous when the field is very large, which means keeping track of the probability mass of the whole field is computationally heavy. The particle filter can focus the computational resource (particles) to the regions of high probability, thus reducing the computational burden. But it causes inaccuracy since the regions of low probability mass are likely to be derived of particles. Since in this simulation the field is of medium size, we use the histogram filter to implement the Bayesian filter.

Thanks to this comment, we realize that we did not explain this clearly in the manuscript. So we have added some description to make readers better understand our choice of implementation methods. We include them here for your reference:

*“”*

1. **The paper organization is good. The presentation is ok, but there are some typos which should be corrected.**

**Response:**

Thank you for the suggestion. We have carefully checked the manuscript and fixed the typos. Some fixed typos include: xxx (to be listed after doing the last-round proofreading).

# Reviewer #2

1. **The paper is technically sounds. However, most of the technical points presented are not author’s original idea or a direct consequence of work presented in this paper. For example, theorem 1 and corollary 1 is not a consequence of the work presented in this paper and is direct result from graph theory. Similarly, theorems 2 and 3 are of little trivial and can’t be suggested as major findings under proposed work.**

**Response:**

The authors would like to thank the reviewer for the comment. The main motivation of this work is developing a distributed Bayesian filter (DBF) that relies on sensor measurements dissemination among UGVs under dynamically changing interaction topologies. We propose the Full-In-and-Full-Out (FIFO) protocol and the FIFO-based DBF (FIFO-DBF) to achieve this goal, which composes the main technical contributions of this work. The theorems presented in this work are derived from the FIFO protocol and FIFO-DBF, and they theoretically analyze the performance of FIFO-DBF. We explain these theorems in more details below.

First, Theorem 1 and Corollary 1 are the consequence of FIFO. If we follow the traditional method that each UGV only sends the current sensor measurement to neighboring UGVs without the use of FIFO, it can happen that two UGVs may never exchange their sensor measurements, even if there exists a path connecting them. It is only by using FIFO to make each UGV send all the received sensor measurements to neighboring UGVs that Theorem 1 holds. Besides, by using Theorem 1 and Corollary 1, we are able to prove the consistency of the proposed FIFO-DBF (Theorems 4 and 5). This is because the use of the law of large numbers in these two theorems relies on the condition that the sensor measurements of all UGVs are constantly received (within finite time intervals) by each UGV. Theorem 1 and Corollary 1 guarantee the satisfaction of this condition.

The previous version of the paper has not clearly presented this point. So we add the following remark in Section 3 in the revised manuscript:

*“Theorem 1 and Corollary 1 are the consequence of FIFO and the use of the communication buffer (CB). In fact, if we use the traditional methods that each UGV only sends the current sensor measurement to neighboring UGVs without the use of CB, it can happen that two UGVs may never exchange their sensor measurements, even there exists a path connecting them. The condition of frequently jointly strongly connectedness is crucial for guaranteeing the consistency of the distributed Bayesian filter, as shown in Section 5.”*

Second, Theorems 2, 3 and Corollary 2 analyzes the trimming of communication buffers (CBs) in the FIFO and its effects on the result of FIFO-DBF. To be specific, Theorem 2 guarantees that using the track list can reduce the communication burden while not affecting the result of FIFO-DBF. Theorem 3 and Corollary 2 quantitatively analyze the communication burden that FIFO will incur. These theorems and corollary justify the use of track list, which in turn makes the proposed FIFO-DBF practically useful, since the communication burden of FIFO is proved to be upper bounded.

To clarify this point, we have included the following sentences in Section 4.2 in the revised version:

*“The following theorem describes when CBs get trimmed, and it provides an upper bound of the communication burden that FIFO will incur. A detailed complexity analysis of FIFO-DBF is presented in Section 4.3.”*

1. **The simulation results are good but however, they cant be considered to be the main results of this paper and thus the technical part of the paper needs to be revised for better chances of acceptance.**

**Response:**

Thank you for the comments. We realize that we did not clarify our technical contributions in the previous submission. Here we would like to explain these contributions in more details:

First, the proposed FIFO protocol for measurement exchange is a novel design for distributed filtering. Previous works usually assume a fixed, strongly connected interaction topology. We propose FIFO so that the measurement dissemination-based distributed filtering can be extended to a class of dynamically changing networks while avoiding the out-of-sequence measurement issue.

Second, we propose the *frequently jointly strongly connectedness* condition of the dynamically changing networks and prove that such class of dynamically changing networks can ensure the dissemination of UGVs’ all sensor measurements in the network, and guarantee the consistency of FIFO-DBF. Though the counterpart concept, called the frequentlyjointly connected undirected networks, was proposed in some early work[[1]](#footnote-1) for the network consensus problem, it is discovered in this work that the extension of such network condition to directed networks is critical for the measurement dissemination-based distributed filtering. We further analyze the communication burden incurred by FIFO.

Third, FIFO-DBF is different from the traditionally used DBF in two ways: (1) The design of the track list is novel and crucial to making FIFO-DBF practically applicable. In fact, the track list significantly reduces the communication burden by keeping a finite size of the communication buffer (CB), proved by Theorem 3 and Corollary 2. In addition, the track list ensures that trimming the CB does not affect the filtering result of FIFO-DBF (Theorem 2). (2) FIFO-DBF keeps an intermediate PDF, called the *stored PDF* (defined in Section 4.1), which is obtained by fusing the sensor measurements of all UGVs up to a certain time t. The use of the stored PDF significantly reduces the computational complexity of FIFO-DBF and the local storage of sensor measurements.

We add the following paragraph in the Introduction section to clarify the contributions of this work:

*“The main focus of the paper is proposing a distributed Bayesian filtering (DBF) method that only uses the measurement dissemination for a group of networked UGVs with dynamically changing interaction topologies. In our previous work, we have proposed a Latest-In-and-Full-Out (LIFO) protocol for measurement exchange and developed a corresponding DBF algorithm. However, it only applies to static targets with simple binary sensor model. In this work, we substantially extend the previous work and make the following contributions:*

*(1) We introduce a new protocol called the Full-In-and-Full-Out (FIFO) that allows each UGV to broadcast a history of measurements to its neighbors by using single-hopping, enabling the tracking of moving targets with general sensor models under time-varying topologies. The use of FIFO also avoids the OOSM issue.*

*(2) We propose the frequently jointly strongly connectedness condition of the network and prove that, under this condition, FIFO can disseminate measurements over the network within a finite time.*

*(3) We develop a FIFO-based distributed Bayesian filter (FIFO-DBF) for each UGV to implement locally. The track list is proposed to reduce the computational complexity of FIFO-DBF and the communication burden.*

*(4) We prove the consistency of FIFO-DBF: each UGV's estimate of target position converges in probability to the true target position asymptotically when the network is frequently jointly strongly connected.”*

We also add the sentences to explain our contributions in other parts of the manuscript, including:

*“The use of FIFO allows us to extend measurement dissemination-based distributed filtering to dynamically changing networks while avoiding the out-of-sequence measurement issue”* in Section 3.

*“The counterpart definition for undirected graphs is given in [11]”.* in footnote 2.

1. **The crux of authors Bayesian filter is determined by equation 3-4. As this forms the main technical background of the method presented in the paper, it is advised to present a better explanation of the equation 4, ie., when does it hold—only if the measurements are iid? If that is the case, then this should be explicitly stated and should be an important assumption of the proposed work.**

**Response:**

Thank you for the suggestion. We have further explained equation 4 and added a commonly adopted assumption to make equation 4 hold: each UGV’s current sensor measurement is assumed to be conditionally independent from its previous measurements and also from other UGVs’ sensor measurements given the target and UGV’s current positions. We include the newly added paragraph for your reference:

*“Here we have utilized the commonly adopted assumption [17, 20, 31] in the distributed filtering literature that the measurements of each UGV at current time are conditionally independent from its own previous measurements and the measurements of other UGVs given the target and the UGVs’ current positions. This assumption allows us to simplify as in Eq. (4a) and factorize as in Eq. (4b).”*

1. **The main result of the paper is theorem 4 and 5 which present consistency of the proposed algorithm. However, they need better explanation—please expand on how the law of large numbers yields equation 8a (It is also advised to expand equation 6 i.e., how to obtain the batch form of the DBF at kth step).**

**Response:**

Thank you for this important comment. The batch form (equation 6) can be derived by recursively applying the updating step (equation 4). We have added the derivation steps in the manuscript and we include them here for your review:

*“The DBF can be transformed into the batch form by recursively applying equation 4 from k to the initial time 1 (back in time):*

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*”*

The equation 8 uses the law of large numbers. Notice that the sensor measurement is a random variable with the distribution . Based on Theorem 1 and Corollary 1, sensor measurements from all UGVs can be constantly received by each UGV, thus the law of large numbers can apply to obtain the asymptotic value of the summation as shown in equation 8. We have added the following sentences in the revised manuscript to help clarify the equation 8:

*“*

*Note that the sensor measurement is a random variable drawn from the underlying distribution associated with the sensor model, i.e., . Therefore* *is a random variable associated with. Due to the finite delay of measurement arrival (Corollary 1), we can use the law of large numbers to study the asymptotic behavior:*

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*”*

1. **The proof of theorem 5 is rather ad-hoc and needs more explanation and needs to be precise.**

**Response:**

We appreciate the reviewer for this comment. In the previous submission we did not give a clear derivation. In this revised version we rigorously proved Theorem 5. Especially, we have added the following paragraphs:

*“*

*The only difference from Eq. (6) is that in Eq. (11) varies as the UGV moves. Similar to Eq. (7), we obtain*

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*where the second summand corresponds to the sensor positions that are in the finite-measurement spots set, and the third summand the positions in the infinite-measurement spots set. By referring to Eq (8), it is straightforward to know*

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*since only finitely many observations associated with sensor positions in are obtained but infinitely many observations associated with sensor positions in are received. the rest of the proof is similar to that of Theorem 4.”*

We hope this revision will improve the proof of theorem 5.

1. A. Jadbabaie, J. Lin, and A. S. Morse, “Coordination of groups of mobile autonomous agents using nearest neighbor rules,” IEEE Transactions on automatic control, vol. 48, no. 6, pp. 988–1001, 2003. [↑](#footnote-ref-1)